RESEARCH TRIANGLE NANOTECHNOLOGY NETWORK

Don’t Fear the Sphere

Overview
Students will investigate the volume of spheres that are scaled from nano to macro. They will investigate the impact of a sphere’s radius to the scale of the sphere's volume. In this three part lesson, students will 1 - order different spheres of known solids by size using educated guesses, 2 - calculate volume or missing radii of each sphere, and 3 - compare the scales of each sphere’s radius and volume.

Key Search Words
NC Math 3, geometry, volume, scale, scale factor, sphere, pyramid, prism, radius, diameter, order, modeling

Learning Objectives
- Students will use geometric formulas to calculate the volume of solids 80% of the time.
- Students will use geometric formulas to calculate a dimension (such as a radius or a side) 70% of the time.
- Students will be able to determine the scale factor of a three dimensional shape given the change of one-, two-, or three dimensions 90% of the time.

Curriculum Alignment
- Understand solving equations as a process of reasoning and explain the reasoning. (NC.M3.A-REI.1)
- Explain volume formulas and use them to solve problems. (NC.M3.G-GMD.3)
- Apply geometric concepts in modeling situations.(NC.M3.G-MG.1)

Classroom time required
Warmup (10 minutes) - optional
Card sort by whole class with discussion (15 minutes)
Complete handout (25 minutes)
Exit Ticket (10 minutes)

Materials & Technology
- Students in no more than nine groups.
- Calculators
- Pencils (for students to write on handout)
- Cards of sphere images (slides 2-10)
- A method to display the cards (i.e. tape for wall display, magnets for board display, clothes pins for string display)
- Don’t Fear the Sphere Handout
- Exit Ticket (see **Assessment/Check for Understanding**)

Optional materials:
- Warmup (see **Teacher Preparation Activity)**
- Soccer balls
- Ping pong balls
- Magnetic metal balls (5 mm)
- Rulers
- Measuring tape
- Metric Cards (slides 11-15)

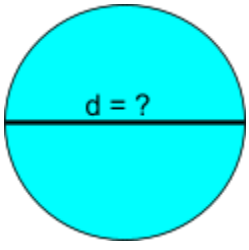
Safety
If optional materials are used, remind students to not throw or bounce balls in the classroom.

Teacher Preparation for Activity

1. Prepare the location you will have the cards displayed (board? wall? string?)
2. Print out the sphere cards (one set)
3. Print a class set of the handout (make sure you do NOT print the Teacher Answer KEY)
4. If you plan to have extension activities, have the website links on your class website for students to access as needed. (See Supplemental Resources)

Student Preparation for Activity

Possible Warm-up:

1. What is the diameter of a circle that has an area of 40π cm^2?  	2. What is the volume of a men's basketball if its circumference is 29.5 inches? (Volume of a sphere is $\frac{4\pi r^3}{3}$) 

Answers: 1. **diameter = 12.65 cm** 2. r = 4.7 in , **volume = 434.9 in^3**

Procedure

1. Warmup (optional)
    a. Have students complete the answers to the warmup at the beginning and review answers.
2. Card Sort
    a. Prepare the location in the room where the sphere images will be posted on a "scale line" (about 12 feet of wall space).
    b. Organize students into 9 or less groups.
    c. Pass out the sphere images so they are all passed out (1 or 2 images per group).
    d. Starting with one group:
        i. Have students share the image that is posted on their card.
        ii. Ask students if they remember the name of the shape (sphere).
        iii. Ask them if they think it is large or small compared to other spheres. They can peek at other cards to help them with the relative sizes.
        iv. Ask them to place their card on the wall where you had set up the "scale line".
        Repeat this step until all of the cards are posted along a continuum (smallest to largest).
    e. Ask students what features of the spheres do they consider when comparing their sizes. (possible answers - diameter, volume, circumference)
    f. Ask students if any of them would like to make any changes.
    g. If you see that any cards are not in the correct order, make the correction.
    h. Once all students agree with the order, mention the idea of scale - some are measured in a scale larger than kilometers, some in meters, some in millimeters; and even nanometers. (as notated on handout)
    i. Optional - pass out the units cards and have them post them on the "scale line" for them to delineate the images.
3. Handout
    a. Pass out the handout. Remind students the formula used to calculate volume of a sphere $\frac{4\pi r^3}{3}$.
    b. Students might need help to determine the radius from a given volume.
    c. Once students complete the main table, have students choose two spheres and compare the radius of larger to the radius of the smaller. Then have them compare the volume of the larger to the volume of the smaller.
    d. Check if they see a relationship between both ratios. Students should be able to relate the ratio of two spheres radius to volume - it is **cubed**.
4. Exit Ticket

a. Hand out the Exit Tickets to each student.
b. You can prompt students by reading the last sentence of each question.

Differentiation

Extensions

- Metric units - students can place metric units on the scale line.(see slides 11-15)
- Application of volumes of other solids (pyramids and prisms).
- Relate objects of similar size - students can name other solids of similar sizes (i.e red blood cell; MMA sporting ring)

Supports

- Calculator use - post steps on order of values on how to calculate volume (i.e. on a TI calculator - students will need to put parentheses if having more than one number in the divisor)
- Have actual models of spheres to help with concept of size

Assessment/Check for Understanding

Exit Ticket	
1. Buckyballs are used by chemists to make some nano-sized structures. A buckyball is a "soccer-like" molecule of carbon atoms. A handheld model is being created by a chemist as a demonstration prop. If the radius of an actual buckyball is scaled up by a factor of one billion, what is the scale factor of the volume increase?	2. As the warm up mentioned, the circumference of a men's basketball is 29.5 inches. If the circumference of a women's basketball is 28.5 inches what is the scale factor of the volume of the men's basketball to the women's basketball?

Exit Ticket - TEACHER ANSWER KEY	
1. Buckyballs are used by chemists to make some nano-sized structures. A buckyball is a "soccer-like" molecule of carbon atoms. A handheld model is being created by a chemist as a demonstration prop. If the radius of an actual buckyball is scaled up by a factor of one billion, what is the scale factor of the volume increase? radius scale factor: 1,000,000000 = 10^9 volume scale factor:$(10^9)^3$ = 10^9 x 10^9 x 10^9 = **10^{27}**	2. As the warm up mentioned, the circumference of a men's basketball is 29.5 inches. If the circumference of a women's basketball is 28.5 inches what is the scale factor of the volume of the men's basketball to the women's basketball? circumference is one dimension: circumference scale factor is $\frac{29.5}{28.5}$= **1.035** volume is 3D: volume scale factor is 1.035^3 = **1.109**

Supplemental resources

Visualizations of comparing size of nanoscale:

- https://www.azonano.com/article.aspx?ArticleID=1780
- https://www.nano.gov/nanotech-101/what/nano-size
- https://nanohub.org/resources/26676/download/Int_Scale_PK12_PG.pdf

Metric notation for meters https://midimagic.sgc-hosting.com/siscale.htm
Teachers can use the table in this website to help students use objects to represent different metric units.

Explanation of Buckyballs - the nano structure made of carbon https://www.popsci.com/buckyball-magic-molecule/

Author comments

- Students tend to have the most difficulty with volume of spheres compared to volume of prisms or pyramids. So, focusing on spheres help students internalize the volume formula which tends to be the most frequently forgotten formula.
- The natural progression of the lesson is to continue with volumes of other solids such as prisms and pyramids. Make sure you include how to find one dimension from the volumes as well as calculating volumes from one dimension.

Sources

Lesson was adapted from Jones, M.G., Taylor A.R. & Falvo, M.R. (2009). *Extreme science: From nana to galactic.* NSTA Press.

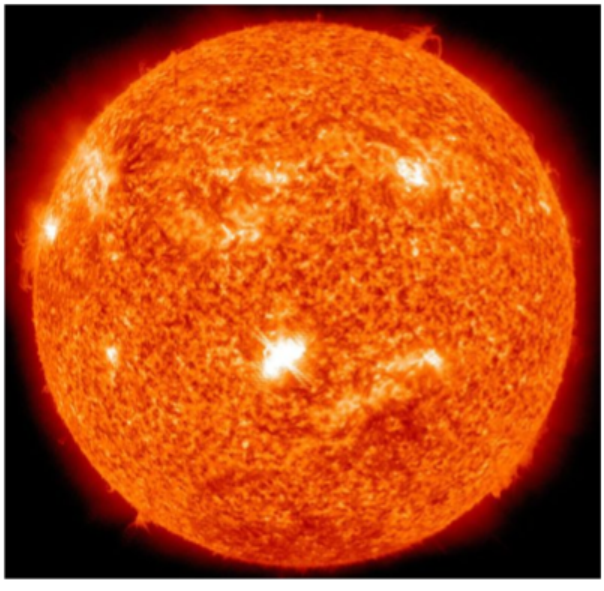
Appendices

Image cards of spheres
Handout (see next page)

Don’t Fear the Sphere
Scaled Spheres cards

Hand out these images in a random order and have students place in on a scale in order by diameter

Sun



https://education.nationalgeographic.org/resource/sun

PHOTOGRAPH COURTESY NASA/SDO/AIA

Earth

https://www.space.com/17638-how-big-is-earth.html
Image credit: NASA/JPL

Epcot's Geodesic Globe

Soccer Ball

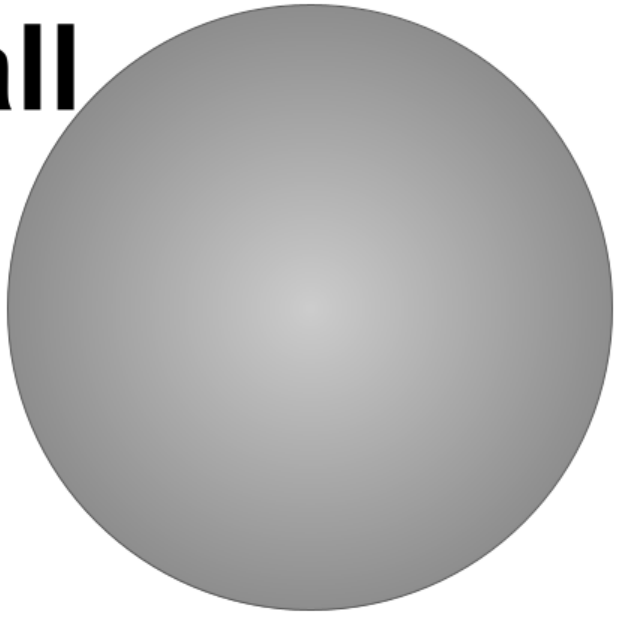
https://commons.wikimedia.org/wiki/File:Football_(soccer_ball).svg
http://openclipart.org/media/files/flomar/6069, CC0,
https://commons.wikimedia.org/w/index.php?curid=3598190

Ping Pong Ball

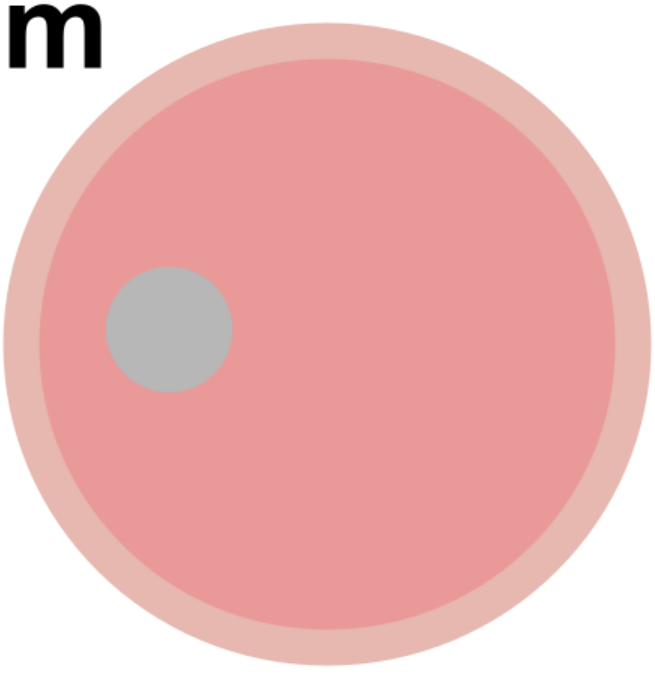
https://franklinsports.com/games/indoor-games/table-tennis

Magnetic Ball

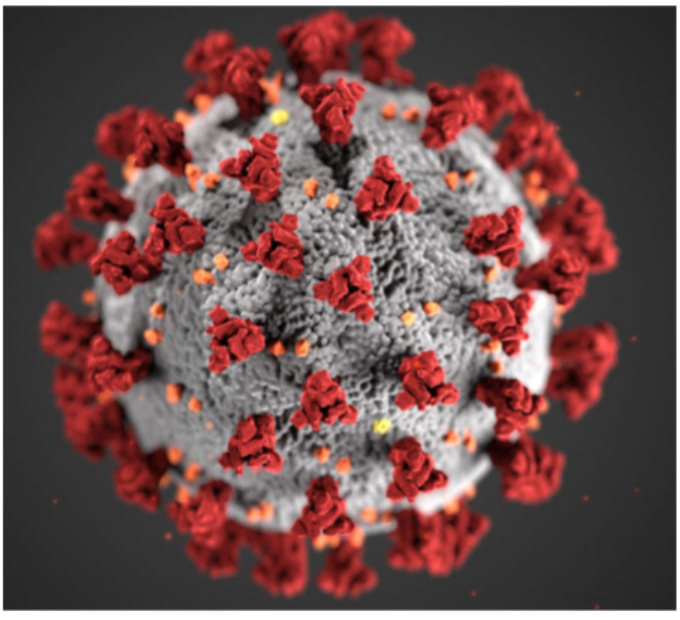
(5mm diameter)



Human Ovum (Egg Cell)

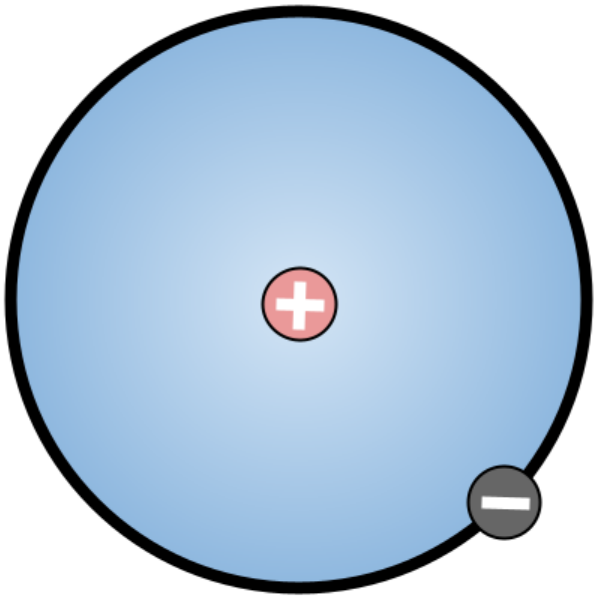


SARS-CoV-2 (COVID-19)



https://www.cdc.gov/dotw/covid-19/index.html

Hydrogen Atom



Optional step:

Cut out each of the “boxes” containing the metric size. After the students place the images in order of smallest to largest, have them place these units on or between the images.

Gigameter (1 Gm) 10^9 meters	**Megameter (1 Mm) 10^6 meters**

Kilometer
(1 Km)
10^3 meters

Meter
(1 m)
10^0 meters

Centimeter
(1 cm)
10^{-2} meters

Millimeter
(1 mm)
10^{-3} meters

Micrometer
(1 um)
10^{-6} meters

Nanometer
(1 nm)
10^{-9} meters

Picometer
(1 pm)
10^{-12} meters

Terameter
(1 Tm)
10^{12} meters

Don't Fear the Sphere **Name _____**

Find the missing radius or volume. Put everything in terms of meters.

Object	**Radius (in meters)**	**Volume** (in meters^3) Keep in terms of π $V_{sphere} = \frac{4\pi r^3}{3}$
Sun	695,700,000 m (695,700 Km)	
Earth	6,371,000 m (6,371 Km)	
Epcot's Geodesic Sphere		21084π m^3
Soccer Ball	.11 m (110 mm)	
Ping Pong Ball		$.0000107\pi$ m^3
Magnetic Ball	.0025 m (2.5 mm)	
Human Ovum (Egg Cell)		$1.67 \times 10^{-13}\ \pi$ m^3
SARS-CoV-2 (COVID-19)	.00000005 m (50 nm)	
Hydrogen Atom	.000000000125 m (.125 nm)	

Select two spheres: (possible choices) _____ and _____		Select two spheres: _____ and _____	
Ratio of radii _____ =	Ratio of volumes _____ =	Ratio of radii _____ =	Ratio of volumes _____ =

What is the relationship between the ratio of the radii and the ratio of the volumes?

Conclusion of scale factor→ if the radius scale is ____, then the volume scale is ____.

Don't Fear the Sphere **TEACHER ANSWER KEY**

Find the missing radius or volume. Put everything in terms of meters.

Object	**Radius (in meters)**	**Volume** (in meters^3) Keep in terms of π $V_{sphere} = \frac{4\pi r^3}{3}$
Sun	695,700,000 m (695,700 Km)	4.489569993 x 10^{26} π m^3
Earth	6,371,000 m (6,371 Km)	3.447954704 x 10^{20} π m^3
Epcot's Geodesic Sphere	25.1 m	21084.33 π m^3
Soccer Ball	.11 m (110 mm)	.00177467 π m^3
Ping Pong Ball	.02 m (20 mm)	.0000107 π m^3
Magnetic Ball	.0025 m (2.5 mm)	2.083 x 10^{-8} π m^3
Human Ovum (Egg Cell)	.00005 m (.05 mm)	1.67 x 10^{-13} π m^3
SARS-CoV-2 (COVID-19)	.00000005 m (50 nm)	1.67 x 10^{-22} π m^3
Hydrogen Atom	.000000000125 m (.125 nm)	2.6 x 10^{-30} π m^3

Select two spheres: (possible choices) __Ping Pong Ball____ and _Magnetic Ball_		Select two spheres: (possible choices) __Ovum___ and ____SARS-CoV-2_____	
Ratio of radius	Ratio of volumes	Ratio of radius	Ratio of volumes
$\frac{.02}{.0025} = 8$	$\frac{.0000107\pi}{2.083\ x\ 10^{-8}\pi} = 513.7$	$\frac{.00005}{.00000005} = 1000$	$\frac{1.67x\ 10^{-13}\pi}{1.67x\ 10^{-22}\pi} = 1000000000$

For each sphere, what is the relationship between the ratio of the volume to the radius?

$\frac{513.7}{8} \approx 64 = 8 * 8$ $\frac{1000000000}{1000} = 1000000 = 1000 * 1000$

Conclusion of scale factor→ if the radius scale is _**x**_, then the volume scale is _**x^3**_