Why a Gecko Can Scale a Vertical Surface

Exploring Exponential Growth of the Surface Area to Volume Ratio

Overview
This lesson is designed to model the change in surface area with respect to volume while also describing the relevance and impact of the surface area to volume ratio. The lesson begins with an activity that has students create an exponential growth equation and graph through recognizing a pattern in blocks growth. Students are asked questions about the blocks' behavior which helps them conceptualize ideas like growth rates and the behavior of exponential functions. Next, students watch a teacher demonstration designed to connect the previous activity to the context of a growing surface area to volume ratio. Students then spend a little time learning about the structure of surfaces at the nanoscale, size-dependent properties, and how both are impacted by the growing surface area to volume ratio.

Key Search Words
Math 1, exponential growth, recognizing patterns, creating equations, modeling

Learning Objectives
- Students should be able to recognise exponential growth and decay
- Students should be able to create equations to model exponential growth
- Students should gain a conceptual/ visual understanding of how exponential equations grow
- Students should understand why small things have a greater surface area to volume ratio
- Students should understand the importance of surface area to volume ratio in nanotechnology

Curriculum Alignment
NC.M1.A-CED.1

Create equations and inequalities in one variable that represent linear, **exponential**, and quadratic relationships and use them to solve problems.

NC.M1.A-CED.2

Create and graph equations in two variables to represent linear, **exponential**, and quadratic relationships between quantities.

NC.M1.G-GPE.4 (optional)

Use coordinates to solve geometric problems involving polygons algebraically

- Use coordinates to compute perimeters of polygons and **areas of triangles and rectangles**.
- Use coordinates to verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.

Classroom time required
Lesson time can vary from **50 minutes to 1 hour and 40 minutes**; be sure to consider how much time you have and look at the suggested times for each activity.

Activity/ Handout: 20-30 minutes
Discussion: 2-3 minutes
Teacher demonstration: 10-15 minutes
Nanoscale video: 3 minutes
Guess the object/ Discussion: 10 minutes
Size dependent properties slides: 5-10 minutes
Optional- Concluding activity: 20-30 minutes

Materials & Technology

- Access to google slides (for the teacher)
- Printer access
- Writing utensils for each student
- Calculators for each student (scientific calculators or phone calculators are fine)
- Clay block for teacher demonstration (must be a cube)
- A tool to divide the block
- Projector (for teacher demonstration)

Optional for Bonus activity

- clay/ playdough- equal amounts per kid
- Laminated graph paper (preferably with labeled axes)
- expo markers

Safety

During the teacher demonstration, you will need to cut a clay block. While a variety of tools can be used to make a clean cut, a knife may be the most readily available tool. If using a knife or any other sharp object, please be careful and take appropriate precautions. Note: Students will not be handling any dangerous materials or tools.

Teacher Preparation for Activity

Prior to implementing this lesson teachers should:

- Review the teacher slides
- Review the student handout/ answer key
- Print copies of the student handout
- Review, and if necessary, practice the teacher demonstration
- Read pages 2- 3 and "Why Do the Greater Surface Area to Volume Ratios Make a Difference?" (page 7) of the following document https://nanosense.sri.com/activities/sizematters/properties/SM_Lesson3Student.pdf
- *Optional*- Review the optional concluding activity. Note: this activity has no handout but does include a discussion slide

Student Preparation for Activity

Prior to this lesson students should know what an exponent is as well as basic exponent rules. For example, students should be able to easily find that $2^3 = 2 \cdot 2 \cdot 2 = 8$ etc.

Procedure

1. Have the teacher slides ready to go at the beginning of class.
2. To start the day, students will be individually completing a student handout. Pass out copies of the student handout and have students answer the questions.
    a. While students are working, be sure to be available if anyone needs help. This could also be a good time to make sure you are prepared for the teacher demonstration.
3. After students have completed the handout, **wait to collect it**. On slide 3 of the teacher slides, there are some discussion questions to ask students before the teacher demonstration.
    a. What did you learn about the growth pattern in the activity? Answers will vary.
    b. How can we apply this to surface area and volume? Students may not know, that is okay. The teacher demo answers this question.
4. Complete the teacher demonstration. For detailed instructions check the appendix.
5. Complete the discussion questions for the teacher demonstration included on slide 3 as well as below.
    a. As size decreases, what happens to the total surface area? It increases
        i. What happens to the surface area to volume ratio? It increases, because surface area is increasing and volume remains constant. Students may also notice that since each block is an equal size when the total surface area to volume ratio doubles, so does the ratio of individual blocks.
6. Watch the short video linked on slide 4 and answer the related discussion questions.
    a. As we zoom in, how does the structure of the surface change? Talk about the change in structure, for example it appears smooth from a distance but up close we see a pattern.

    b. What does this have to do with surface area? The texture found up close actually means the surface area is greater than we may expect from a distance.
        i. Remember answers will vary. These questions are designed to get students thinking and it is more important to participate than to have the "right answer".
7. Slides 5-13 have a "guess the object" game where the slides switch between a zoomed-in object and what the object looks like to the naked eye. For this activity, ask the students to guess what they think the object is before revealing what it actually is on the next slide. The activity is designed to introduce students to the idea that things may look different than we expect up close.
8. Complete the discussion questions on slide 14. These questions are designed to connect the ideas about how the surface area to volume ratio change can be put to use by taking a closer look at objects.
    a. What do the pictures we just looked at have to do with the surface area to volume ratio? The texture found up close actually means the surface area is greater than we may expect from a distance. Therefore the surface area to volume ratio will also be larger than expected. This can be particularly true when looking at a small section of something.
    b. How do the zoomed in pictures make you think differently about these objects? Answers will vary, no wrong answers here
    c. Were the zoomed in pictures what you expected? yes/ no (again no wrong answers). If students answer no you may want to ask why or what they did expect.
    d. What could this new knowledge of what the surface of objects looks like up close tell us? Answers may vary/ no wrong answers. This new knowledge could affect the surface area to volume ratio like we talked about, how light hits the object like the butterfly video talks about, physical properties etc.
9. Complete the size dependent properties presentation found on slides 15-17. These slides may require more background knowledge for you as the teacher, please see Teacher Preparation for Activity or Supplemental Resources sections to learn more.
    a. Slide 16 has some discussion questions to get students thinking and check for understanding.
        i. What does that mean for nanosized particles? (they have a very large surface area to volume ratio)
        ii. What could this mean for substances with greater surface area to volume ratios? remember this refers to smaller particles (they will behave differently since they will have more particles in contact with a surrounding material and therefore be more subject to different forces than larger objects)
        iii. What does this tell us about reaction time for smaller groups of particles? (reminder we have stated that smaller= greater surface area to volume ratio and having a greater exposed surface area results in a faster reaction) A: it is faster bc of the greater surface area to volume ratio
10. Your lesson is complete: If you have extra time, consider looking more into the examples on slide 18 with your students (it can be as easy as a quick google search or sharing the infographic found under Supplemental Resources about gecko's ability to scale walls)
11. ***Optional***- Slides 18-19 include instructions and discussion questions for a bonus activity to do with your students if desired.

Differentiation

More **advanced students** may work through the activity/ activities faster. Consider focusing on meaningful discussions. While there are some questions provided, students could also come up with additional questions. If students finish early, consider using the optional concluding activity.

Some groups of students may be **less talkative** during discussions. Be sure to encourage participation and emphasize that there is nothing wrong with being wrong, it is okay to have incomplete ideas, and many questions do not have one right answer. Additionally, you may choose to stick to the provided questions, give hints, and more closely guide these students in their discussion.

Assessment/Check for Understanding

Teachers will determine student achievement through understanding shown in the several class discussions, as well as student's success on the student handout (see appendix for the teacher key).

Required resources

Teacher Slides

- The teacher slides are designed to have a presentation to go through with the students. The slide deck includes activities, discussion questions, and a few slides about size-dependent properties.

Growing Blocks Handout (found in the appendix)

- This handout can be printed and distributed to students. The handout is designed to help students create an exponential growth equation to describe a pattern while also helping them visualize what exponential growth (and decay) looks like.

Teacher Demonstration Instructions/ Procedure

- This document contains detailed instructions on how to complete the teacher demonstration. The teacher demonstration is critical to the lesson as it allows students to draw a connection between the handout they have just completed and how surface area grows relative to volume.

Teacher Key (found in the appendix)

- The teacher key contains a completed version of the Growing Blocks Handout.

Supplemental resources

A resource to learn more about size-dependent properties:
https://nanosense.sri.com/activities/sizematters/properties/SM_Lesson3Student.pdf
Suggested reading: pages 2- 3, and "Why Do the Greater Surface Area to Volume Ratios Make a Difference?" (page 7)

Short infographic about gecko's ability to scale walls:
https://www.acs.org/content/dam/acsorg/education/students/highschool/chemistryclubs/infographics/geckos-infographic.pdf

A short video zooming into a blue morpho butterfly's wing in order to show the structure which gives the wings their iridescent properties.
https://www.nisenet.org/catalog/zoom-blue-morpho-butterfly-video

Author comments

The optional activity at the end of the lesson may require more preparation but it could be useful for teachers with longer class periods, teachers looking to tie in an extra learning objective, or teachers with more advanced students. If you are planning on doing this activity, I **highly recommend** taking the option to tie it in through the teacher demonstration as this extra step will give students an example of what they are expected to do.

Sources

NanoSense. (n.d.). *Lesson 3: Unique properties at the nanoscale ... - sri international*. Lesson 3: Unique Properties at the Nanoscale. Retrieved June 28, 2022, from https://nanosense.sri.com/activities/sizematters/properties/SM_Lesson3Student.pdf

National Geographic Society. (2015, August 7). *How the nanoscale measures up*. National Geographic Society. Retrieved June 30, 2022, from https://www.nationalgeographic.org/activity/how-nano-scale-measures/

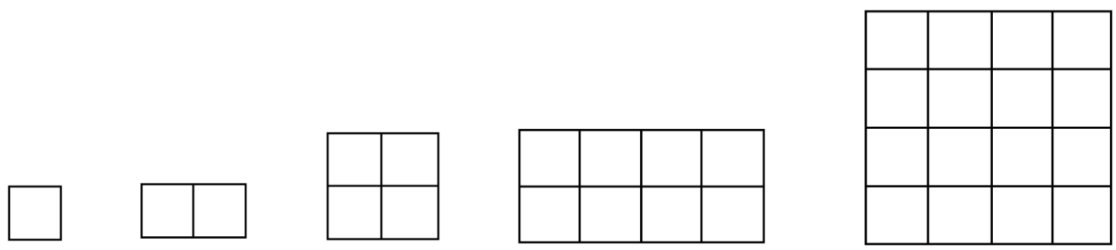
Stevens, S. Y., Sutherland, L. M., & Krajcik, J. S. (2009). *The big ideas of nanoscale science and engineering*. NSTA press.

Appendix

Growing Blocks

Student Worksheet

Answer the questions about the figures below. Make sure to show your work!



(fig. 1) *(fig. 2)* *(fig. 3)* *(fig. 4)* *(fig. 5)*

Draw the 6th figure.

How many boxes are in the 6th figure?

How many boxes are in the 10th figure?

Explain how the figures are growing.

Explain the pattern in the number of boxes between figures.

How many boxes would be in figure 0? Why?

Draw figure 0.

How many boxes would be in the -1st figure? Why?

Write an equation for the number of boxes in the nth figure. Graph your equation.

Use your equation to find the number of boxes in the 72nd figure. *(show your work)*

Use your equation to find the number of boxes in the 45th figure. *(show your work)*

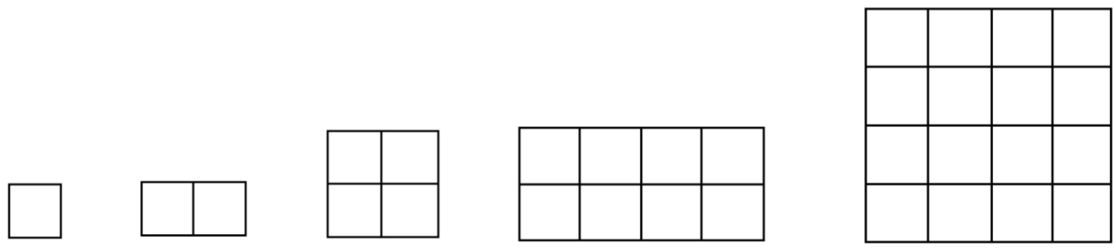
How many boxes would be in figure 4.5?

Challenge: draw the figure between figures 4 and 5. *(Hint: Think figure 4.5)*

Growing Blocks

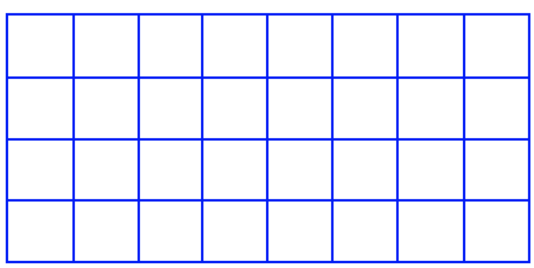
Student Worksheet Teacher Key

Answer the questions about the figures below. Make sure to show your work!



(fig. 1) (fig. 2) (fig. 3) (fig. 4) (fig. 5)

Draw the 6th figure.



How many boxes are in the 6th figure?

$4 \times 4 = 16$ (number of bokes in figure 5)

$16 \times 2 = 32$

Alternatively students could count the boxes using their last answer

How many boxes are in the 10th figure?

Figure 6 = 32 boxes

Fig. 7: 32 x 2 = 64 boxes

Fig. 8: 64 x 2 = 128 boxes

Fig. 9: 128 x 2 = 256 boxes

Fig. 10: 256 x 2 = 512 boxes

Solution methods may vary

Explain how the figures are growing.

The boxes from the previous figure are being duplicated and attached to either the side or the top.

Answers may vary

Explain the pattern in the number of boxes between figures.

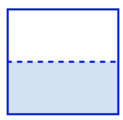
The number of boxes **doubles** between each figure.

How many boxes would be in figure 0? Why?

$1/2$ of a box

Because as figures increase the number of boxes increases by a factor of 2. Therefore going in the opposite direction the number of boxes will decrease by a factor of 2.
Explanations may vary

Draw figure 0.



How many boxes would be in the -1st figure? Why?
1/4 of a box
Because if you go backward from figure 0 by dividing by 2 (or multiplying by 1/2) you will get 1/4th
Explanations may vary

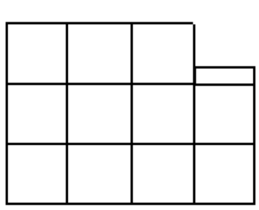
Write an equation for the number of boxes in the nth figure. Graph your equation.
$x = 2^{(n-1)} = \frac{1}{2}(2)^n$
Where x is the number of boxes in the nth figure
Students should also draw a graph their equation. The x axis should be figure number and the y axis will be number of boxes.

Use your equation to find the number of boxes in the 72nd figure. *(show your work)*
$x = 2^{(n-1)}$
$x = 2^{(72-1)}$
$x = 2^{71} = 2.3612 \bullet 10^{21}$

Use your equation to find the number of boxes in the 45th figure. *(show your work)*
$x = 2^{(n-1)}$
$x = 2^{(45-1)}$
$x = 2^{44} = 1.759 \bullet 10^{13}$

How many boxes would be in figure 4.5?
$x = 2^{(n-1)}$
$x = 2^{(4.5-1)}$
$x = 2^{3.5} = 11.314$

Challenge: draw the figure between figures 4 and 5. *(Hint: Think figure 4.5)*



Teacher Demonstration

Materials:
- ➔ Block of clay (can be any size but must be a cube and should be easily manageable)
- ➔ A tool to cut through the clay
- ➔ A projector

Instructions/ Procedure:

1. Find the surface area of the block with the class

a. Keep in mind since you are using a cube you only need to find the length of one side or the area of one side whichever method you prefer.

b. After measuring calculate the surface area

c. If you would like to tie in the bonus activity: Calculate the area of one side by plotting the shape on laminated graph paper, then using the coordinates to solve for the area. (be sure to point out that a cube has 6 identical sides and multiply your area by 6 to get the surface area)

d. Record the surface area that you find

2. Take the block and cut it in half on each side to produce 8 identical smaller blocks

a. Be sure to point out that the total volume remains constant no matter how many cuts are made

3. Find the total surface area of the 8 blocks

a. Find the area of one side (with the option to use graph paper and coordinates)

b. Multiply the area you find by 6 to get the surface area of on cube

c. Multiply the surface area you find by 8 the get the total surface area for all cubes

d. **Record the surface area that you find**

e. Your resulting surface area should be approximately 2 times the original surface area from step 1.

4. Divide one of the 8 blocks the same way as you did in step 2

5. Repeat step 3

a. This time you should multiply by 8 an additional time to account for all 8 blocks created in step 2.

b. **Record the surface area that you find**

c. Your resulting surface area should be approximately 2 times the surface area found in step 3 and 4 times the original surface area from step 1.

6. Ask students what they think would happen if the same procedure was repeated again? (if you divided the “64” blocks each into 8 more)

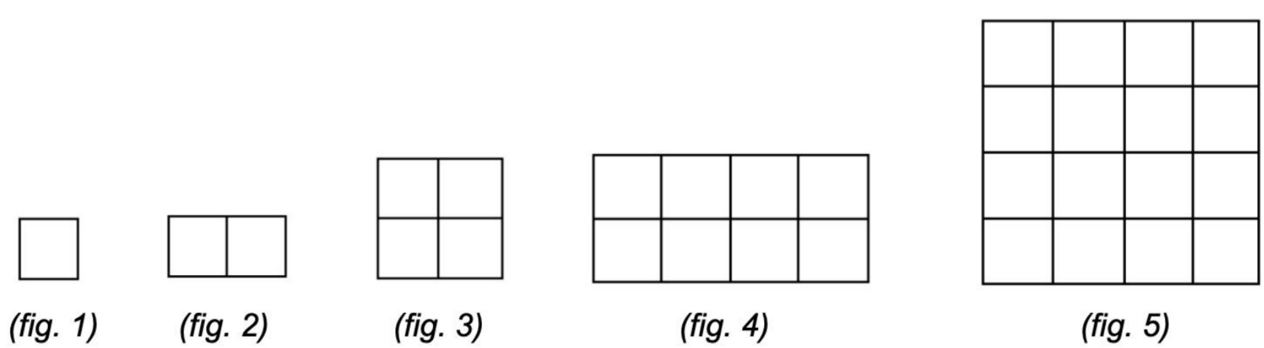
7. Ask students to write an equation that will calculate the surface area over n number of iterations

a. Have students record their answers on a whiteboard and all hold up their equation at the same time

Questions:
- As size decreases what happens to the surface area to volume ratio?

Size dependent properties

Activity (Handout)



Discussion: Activity

- What did you learn about the growth pattern in the activity?
- How can we apply this to surface area and volume?
- Teacher demonstration
  - As size decreases what happens to the total surface area?
    - what happens to the surface area to volume ratio?

What if we zoom in to the nanoscale?

Click below to view a short video zooming into a blue morpho butterfly's wing in order to show the structure which gives the wings their iridescent properties

https://www.nisenet.org/catalog/zoom-blue-morpho-butterfly-video

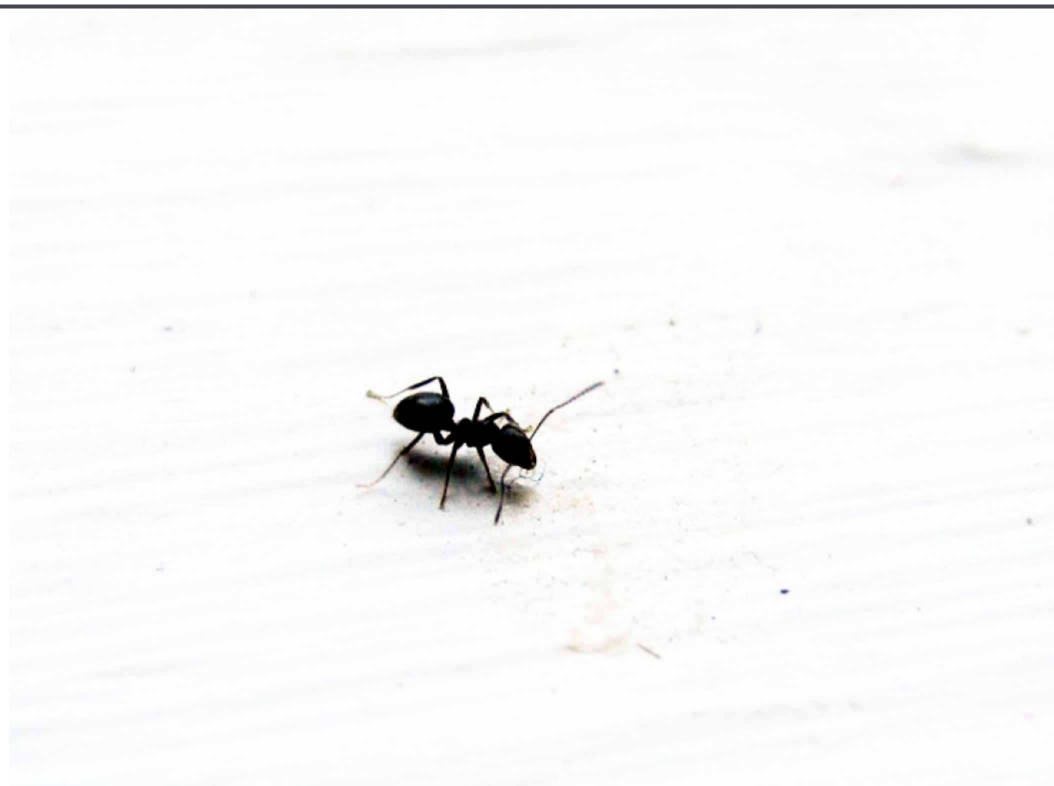
Discussion:

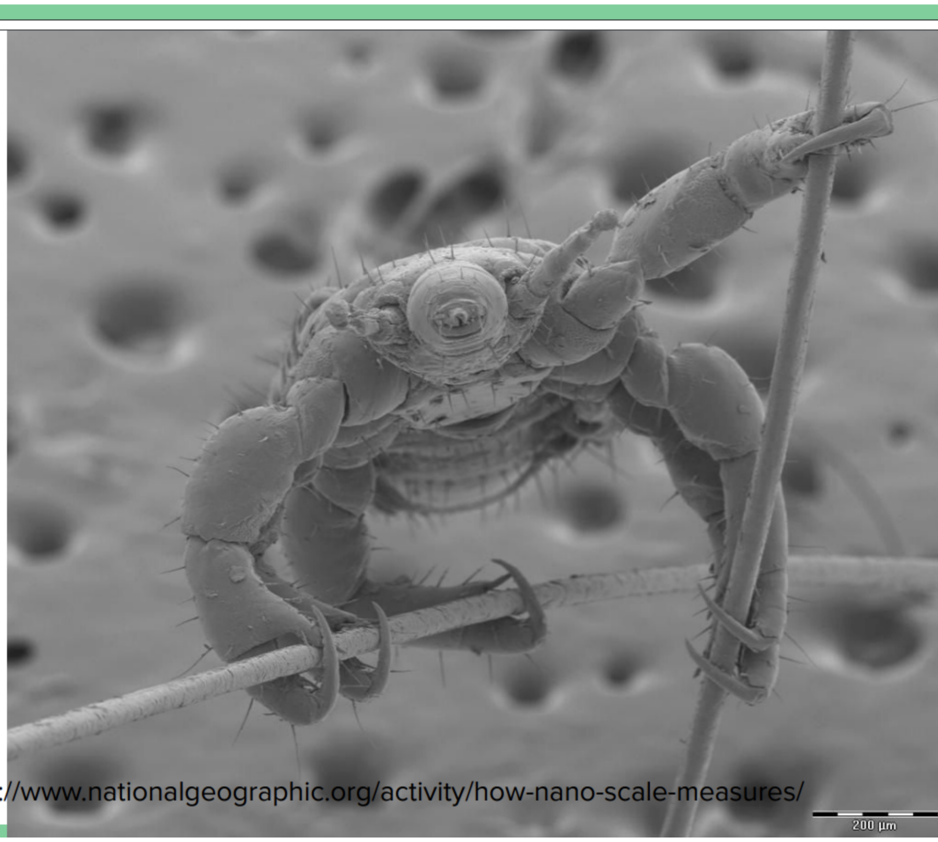
- As we zoom in how does the structure of the surface change?
- What does this have to do with surface area?

Guess the object

https://www.nationalgeographic.org/activity/how-nano-scale-measures/

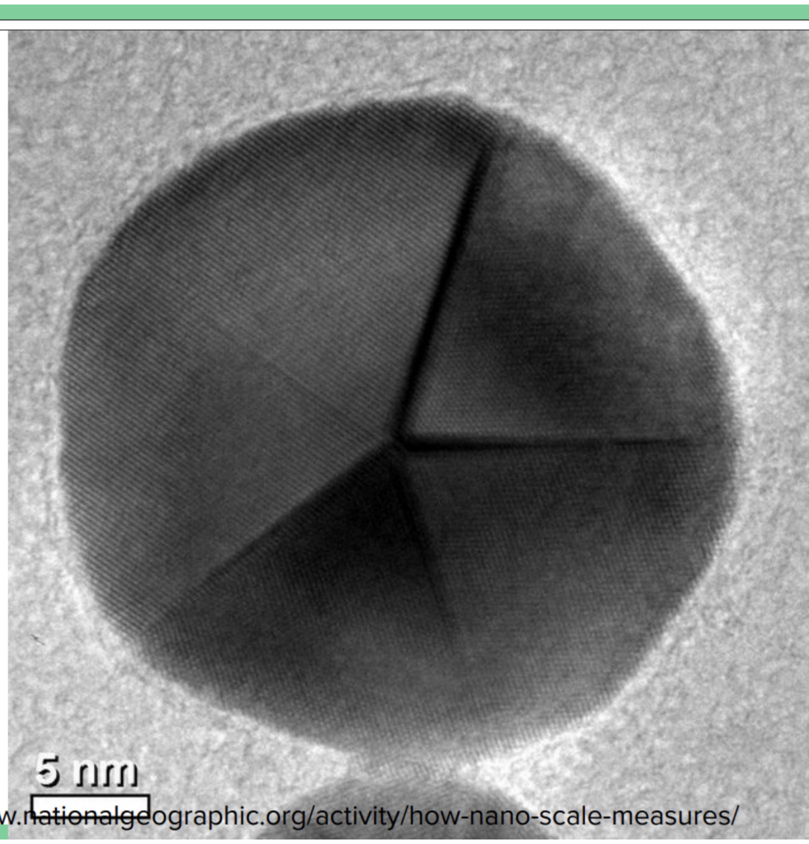
Ant head





https://www.nationalgeographic.org/activity/how-nano-scale-measures/

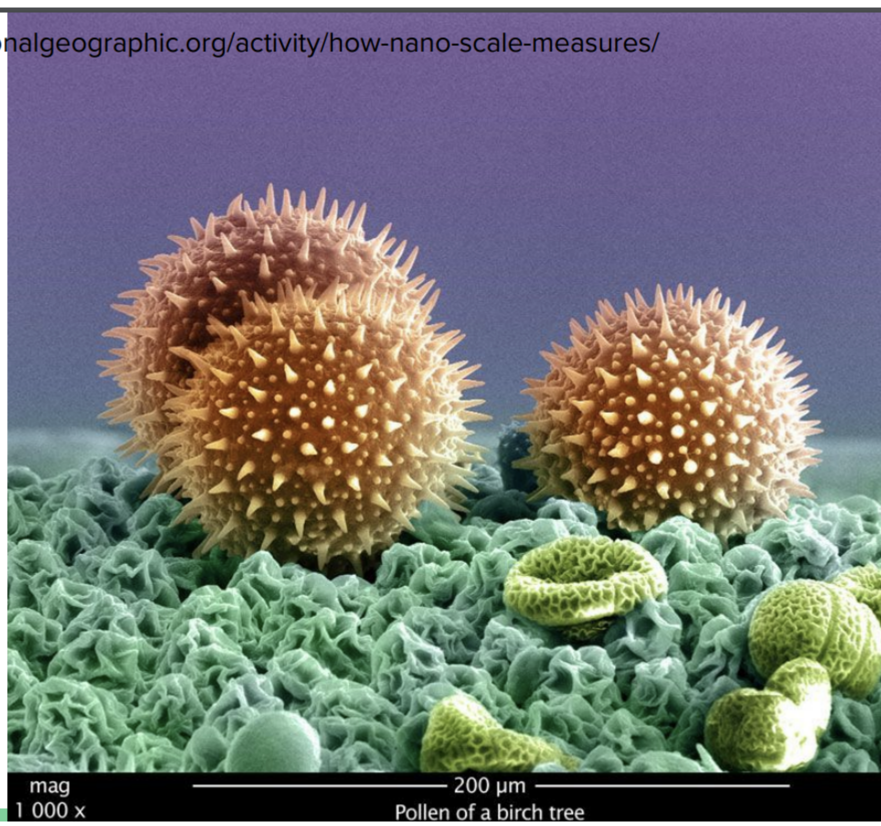
Head louse on human hair



https://www.nationalgeographic.org/activity/how-nano-scale-measures/

Gold Particle

https://www.nationalgeographic.org/activity/how-nano-scale-measures/



Pollen on a birch tree

Discussion

What do the pictures we just looked at have to do with the surface area to volume ratio?

How do the zoomed in pictures make you think differently about these objects?

Were the zoomed in pictures what you expected?

What could this new knowledge of what the surface of objects looks like up close tell us?

Size dependent properties

- Measurements made in labs have previously been limited to averages based on billions and billions of particles
- Tiny amounts of a substance can actually have different properties
- Sand analogy
  - Think about how sand looks from a distance: smooth, uniform color, we might assume each grain of sand has the same properties such as color, texture, and size
  - Now think about zooming in to a few grains of sand: each one can have various colors, textures, and size

https://nanosense.sri.com/activities/sizematters/properties/SM_Lesson3Student.pdf

Size dependent properties continued

- Smaller objects have a greater surface area to volume ratio
  - What does that mean for nanosized particles?
- While inner particles only react with other particles in substance, surface particles interact with surrounding materials. These different materials have different properties and therefore interact differently.
  - What could this mean for substances with greater surface area to volume ratios?
- Since reactions occur where 2 substances meet- max exposed SA = max reactivity
  - What does this tell us about reaction time for smaller groups of particles?

https://nanosense.sri.com/activities/sizematters/properties/SM_Lesson3Student.pdf

Examples

Geckos ability to scale vertical glass walls can be attributed to the high surface area to volume ratio found on their toe-pads, in addition to van der Waals forces. This ability could be harnessed for future adhesive technology

The color of gold changes in small amounts- gold can be red or even purple

- Particles in small amounts absorb/ reflect light differently
- Color actually depends on size and bonding arrangement of particles

https://nanosense.sri.com/activities/sizematters/properties/SM_Lesson3Student.pdf

"Unpacking of Big Ideas in Nanoscale Science" resource

Bonus Activity

Goal: Try to create the largest surface area possible given a set volume

Materials: Clay (or playdough), a laminated sheet of graph paper, and a sharpie for each student

Instructions:

1. Use your clay (or playdough) to create a shape of your choice trying to maximize the surface area. Your shape must have sides (ie not a sphere).
2. Use your graph paper to calculate the surface area of your object by plotting each side on the graph paper.
3. To find the area of the sides you have plotted, first divide your shape into triangles and rectangles (it is okay to estimate), then label important coordinates and calculate the area of all the smaller shapes, finally add these areas together
4. When you are finished record your total surface area

Discussion: Bonus Activity

- What do you notice about the volume?
- What do you notice about the surface area?
  - Surface area to volume ratio?
- Why do you think this matters?
- What shapes have the highest ratio?
  - What about the lowest ratio?